false lock phenomenon, and that high-speed high-accuracy demodulation is achievable by a simple configuration. The influences of some degradation factors to bit-error-rate characteristics are studied experimentally, and their design objectives are stated.

ACKNOWLEDGMENT

The authors wish to thank Dr. H. Kimura, Dr. S. Seki, H. Ishio, and Dr. M. Akaike of the Electrical Communication Laboratories of NTT, and T. Kuroda and Dr. M. Sugiyama of the Nippon Electric Company, Ltd., for their helpful discussions and encouragement.

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Calibrating the Six-Port Reflectometer by Means of Sliding Terminations

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Abstract—**The six-port technique promises to have a major impact on the next generation of automatic network analyzers because complex heterodyne methods may be replaced by simple amplitude detectors. This projection, however, is predicated upon the existence or development of calibration techniques which permit one to conveniently and accurately obtain the parameters which characterize the six-port. This paper describes a number of substantial refinements to a previously described procedure which is based upon the use of sliding terminations.**

I. INTRODUCTION

THE APPEAL OF the six-port measurement concept stems largely from the simplification which it affords in the associated detection circuitry. Instead of complex heterodyne schemes, simple diode, thermoelectric, or bolometric detectors may be used. Because frequency conversion and mixing have been eliminated, practical experience, to date at least, indicates that a high order of stability in the source frequency, e.g., a frequency synthesizer, is not essential (although it certainly may be useful). Because of these simplifications, the six-port technique promises to have a major impact upon the next generation of automatic network analyzers. Morever, recent theoretical work has yielded an improved physical insight into the method so that it may now be applied with greater confidence [1].

Manuscript received May 29, 1978; revised July 31, 1978.
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The projected applications, however, are contingent upon the existence or development of appropriate calibration techniques which permit one to conveniently and accurately obtain the parameters which characterize the six-port. Although the calibration task does not pose any problems of a fundamental character, the supplementary requirements for convenience and speed (while not sacrificing accuracy), together with the general constraints imposed by an automated environment, combine to form the major challenge associated with the method.

As contrasted with the four-port reflectometer (which provides the basis for the existing network analyzers), the six-port reflectometer requires eleven rather than six constants for its calibration. Given this information, one might anticipate that the number of required terminations or standards and thus operator and computational effort would perhaps also be doubled. While this is nominally true of the computational effort, the procedures can still be handled by a desk-top programmable calculator.

Fortunately, this calibration scheme, along with others that have been described [2], is only slightly more involved in terms of required terminations and operator effort than that of a four-port device. When coupled with the projected long-term stability of the calibration results, it appears that the total calibration effort associated with a six-port measuring system may ultimately become substantially less than that for a four-port.

In an earlier paper [3], a calibration procedure was described which was based upon the use of sliding terminations of small and large reflection. Although this method did find some use, the solution called for time consuming iterations which, in the absence of a good initial estimate, might fail to converge to the desired solution. This has been corrected. In particular, the solution now exists in closed form.

II. Theory

The theoretical approach to be followed in this analysis was introduced in the prior paper [3]. It permits the problem to be separated into two distinct parts. In the first, as explained below, the six-port is reduced to an equivalent four-port reflectometer including an associated complex ratio detector. This requires a determination of five of the eleven constants which describe the six-port. The second part of the problem is to determine the six real (or three complex) constants which characterize the equivalent four-port.

This approach has several advantages. 1) It is not necessary to determine all eleven constants simultaneously. 2) A number of solutions already exist for the second part of the problem; these may be applied with little or no modification after a solution to the first part has been obtained. 3) The approach provides a convenient method for exploiting the redundancy which is inherent in the method. Because solutions to the four-port problem already exist, this paper will focus primarily on the six- to four-port reduction.

Let the six-port of interest be represented by Fig. 1. In addition to the four power meters $P_3 \cdots P_6$, incident and emergent waves have been identified at certain ports and labeled a_i and b_i $(i=2,3,4)$, respectively. If by some means one can determine the complex ratio between the emergent wave amplitudes at two of the sidearms, e.g., b_3/b_4, in terms of $P_3 \cdots P_6$, then the desired reduction to a four-port has been achieved.

Now, by definition[1],

$$|b_3|^2 = P_3 \tag{1}$$

$$|b_4|^2 = P_4 \tag{2}$$

so the magnitude of b_3/b_4 is immediately available.

Next, following the general development given in [3], one has

$$P_5 = |Kb_3 + Lb_4|^2 \tag{3}$$

[1]Strictly speaking, proportionality factors should be included. However, since they cancel from the final result, it is convenient to omit them.

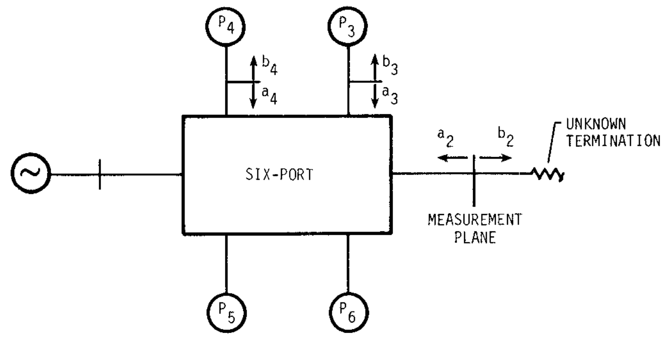

Fig. 1. Given an arbitrary six-port and four power meters, $|b_2|^2$ and a_2/b_2 may be determined at the measurement plane from observations of $P_3 \cdots P_6$.

$$P_6 = |Mb_3 + Nb_4|^2 \tag{4}$$

where $K \cdots N$ are constants, whose values are intrinsic properties of the six-port. Taking the ratios of (1), (3), and (4) to (2) gives

$$|w|^2 = \frac{P_3}{P_4} \tag{5}$$

$$|w - w_1|^2 = \zeta \frac{P_5}{P_4} \tag{6}$$

$$|w - w_2|^2 = \rho \frac{P_6}{P_4} \tag{7}$$

where w is b_3/b_4, w_1 is $-L/K$, w_2 is $-N/M$, ζ is $1/|K|^2$, and ρ is $1/|M|^2$. Here $P_3 \cdots P_6$ are the observed quantities, and w_1, w_2, ζ, and ρ are, for the moment, assumed to be known in terms of the six-port parameters. Thus (5)–(7) may be considered a system of simultaneous equations for w.

The solution to these equations for w is shown graphically in Fig. 2. In particular, the solution is given by the intersection of three circles centered at the origin, w_1, and w_2, respectively, and whose radii are $\sqrt{P_3/P_4}$, $\sqrt{\zeta P_5/P_4}$, and $\sqrt{\rho P_6/P_4}$. In Fig. 2, the argument of w_1 has been arbitrarily assigned the value zero. The justification for this follows in that the arguments of w_1 and w_2 are determined, in part, by the positions of the reference planes in arms 3 and 4.[2] Since these have been introduced only for convenience in the analysis, their locations may be assigned arbitrarily. In what follows, it is convenient to assume that these terminal planes have been chosen such that, as shown in Fig. 2, arg $w_1 = 0$. This solution may be considered a special case of the more general one given in [1] and where the w_i takes the role of the q_i. Here the circles are exactly centered at the w_i, while in the general case this is, at best, an approximation. Moreover, since one of the circles is centered at the origin and another on the real axis, the number of parameters required to specify the w_i, as compared to the q_i, have been reduced from six to three.

[2]This may be recognized as follows. Starting with (3), a shift in the terminal plane in arm 4 must affect arg b_4 while leaving P_5 unaltered. This leads to the conclusion that the change in arg b_4 is accompanied by a change in arg L in the opposite direction. The same must hold in arm 3. This leads to the above result.

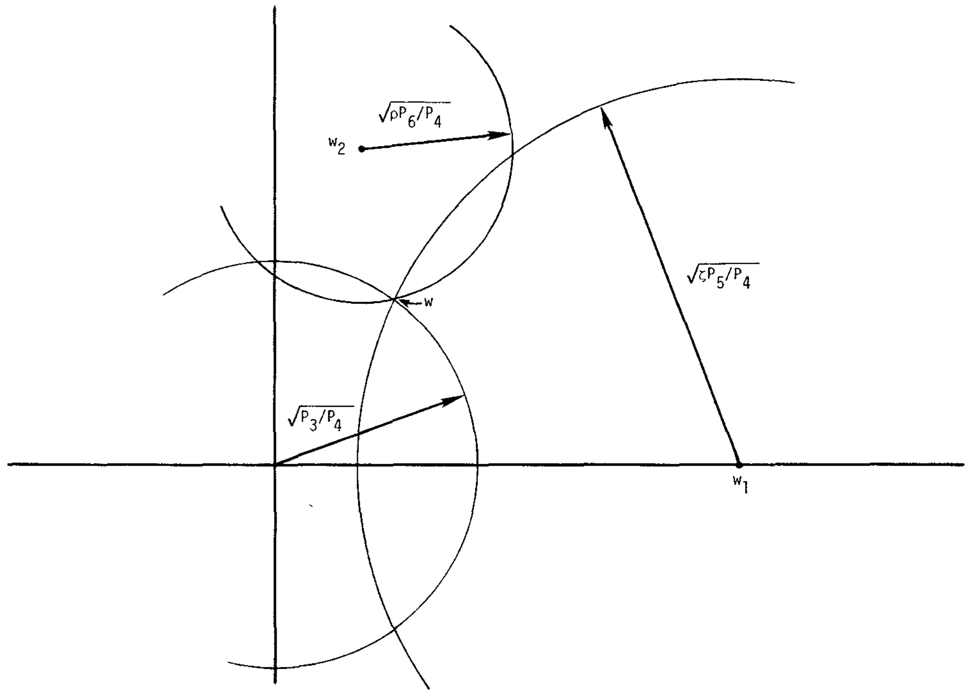

Fig. 2. The complex value of w (which is related to a_2/b_2) is determined by the intersection of these circles.

Before a practical application of the reduction to a four-port can be made, however, it is first necessary to determine the real parameters w_1, ζ, and ρ, and the complex parameter w_2, which together comprise the five constants which characterize this reduction.

Returning to Fig. 2, the circle centered at w_2 must pass through the intersection of those centered at 0 and w_1. Thus, apart from its measurement, P_6 is determined by P_3, P_4, and P_5 to the extent of a choice between two possible values. Since this is true for all values of w, there must be a constraining relationship among P_3, P_4, P_5, and P_6 and w_1, w_2, ζ, and ρ. This may be found by eliminating w from (5)–(7).

The elimination may be achieved as follows. First, the left-hand sides of (6) and (7) are expanded and $|w|^2$ eliminated by (5). This leaves a pair of equations which are linear in the real and imaginary parts of w. Solving for these and setting the sum of their squares equal to P_3/P_4 (cf. (5)) completes the straightforward but lengthy procedure. This gives

$$a\left(\frac{P_3}{P_4}\right)^2+b\zeta^2\left(\frac{P_5}{P_4}\right)^2+c\rho^2\left(\frac{P_6}{P_4}\right)^2+(c-a-b)\zeta\left(\frac{P_3P_5}{P_4^2}\right)+(b-a-c)\rho\left(\frac{P_3P_6}{P_4^2}\right)$$
$$+(a-b-c)\zeta\rho\left(\frac{P_5P_6}{P_4^2}\right)+a(a-b-c)\frac{P_3}{P_4}+b(b-a-c)\zeta\frac{P_5}{P_4}+c(c-a-b)\rho\frac{P_6}{P_4}+abc=0 \quad (8)$$

where

$$a=|w_1-w_2|^2 \quad (9)$$

$$b=|w_2|^2 \quad (10)$$

$$c=|w_1|^2. \quad (11)$$

It is convenient to think of P_3/P_4, P_5/P_4, P_6/P_4 as representing a point in a three-dimensional "P space." Equation (8) then represents a quadric surface in P space. The application of a standard test (see, for example, [6]) shows that this surface is a paraboloid, and which from other considerations must be of the elliptic (rather than hyperbolic) type. Moreover, it can be shown[3] that this surface is tangent to the planes $P_3/P_4=0$, $P_5/P_4=0$, and $P_6/P_4=0$.

The parameters which characterize the paraboloid have now been identified as a, b, c, ζ, and ρ. In a following paragraph, an explicit expression for w (or b_3/b_4) will be developed in terms of these constants and the observed $P_3 \cdots P_6$. The immediate goal is to determine $a \cdots \rho$.

The approach followed in the earlier paper [1] was to observe the $P_3 \cdots P_6$ for five or more arbitrary and unknown terminations. This yields a set of simultaneous equations in $a \cdots \rho$; unfortunately, as inspection of (8)

[3]This may be recognized as follows. If in (6), for example, $P_5=0$, then w is uniquely determined, i.e., $w=w_1$. This, in turn, uniquely determines P_3/P_4 and P_6/P_4 via (5) and (7). Since the plane $P_5/P_4=0$ and the surface defined by (8) have only a single point in common, the plane is evidently tangent to the paraboloid.

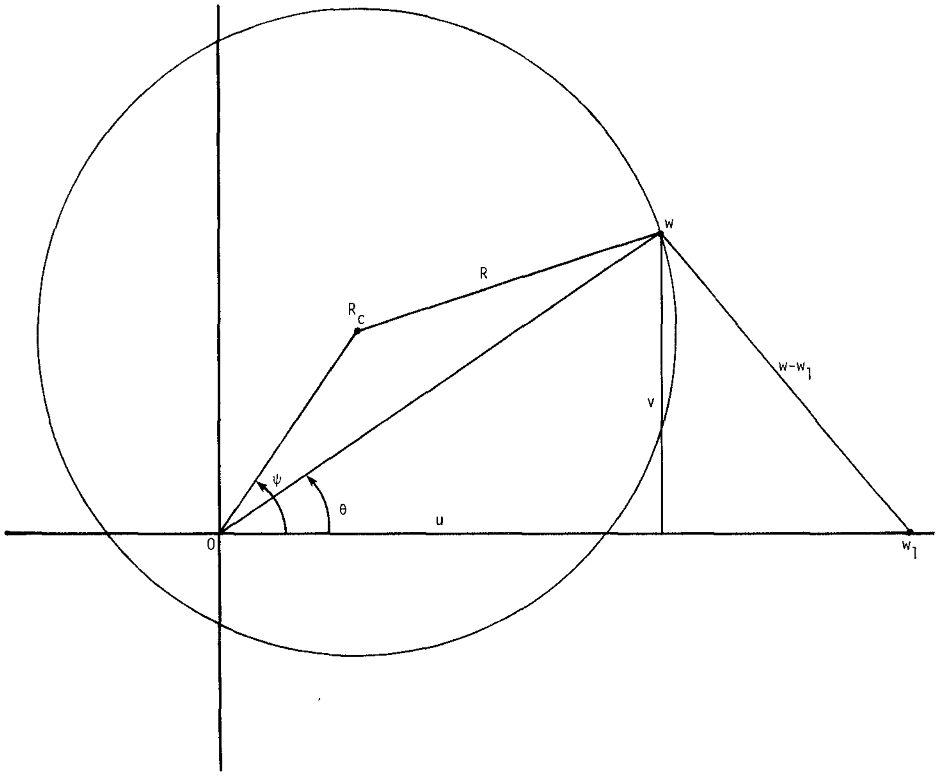

Fig. 3. A sliding short produces a circular locus in the w plane.

shows, they are of the third degree. When a solution is attempted, using standard numerical methods, the iteration tends to be lengthy and may diverge or find a wrong root unless a reasonably good estimate of the solution has first been obtained. The chances of obtaining the desired root is improved by increasing the number of arbitrary terminations and the observed $P_3 \cdots P_6$, but this also increases the iteration time.

If the number of terminations is increased to nine, an alternative approach is possible. In particular, a general quadric surface requires nine constants for its specification, and the observed $P_3 \cdots P_6$ permits their determination via a system of nine linear equations. In principle, this surface would be the paraboloid of interest; because of measurement error, however, the paraboloid and tangency conditions will only be approximately satisfied. This procedure does yield, however, a good starting point for an iterative solution based on (8). In the immediate context, an alternative approach also warrants consideration.

One of the objectives in seeking a solution to this problem is to keep the operator involvement as simple as possible. Stated in other terms, one would like to make the maximum possible use of the data collected in anticipation of determining the remaining parameters which pertain to the equivalent four-port. One of the more attractive four-port calibration methods [4] calls for observing the system response to a single-impedance standard and to three (or more) positions each of a sliding termination and sliding short. Thus it is appropriate to ask what can be determined about the six- to four-port reduction, using these measurements as a starting point.

At the measurement terminals, where the sliding short is connected, let $\Gamma = a_2/b_2$. In the complex Γ plane, the locus of Γ values for the sliding short is a circle, centered at the origin and of a nominal-unit radius. Now, w (or b_3/b_4) is related to Γ via a bilinear transformation whose parameters are those required to describe the reduced four-port. Although these are unknown at this point, a well-known property of any bilinear transformation is that circles are mapped into circles (with straight lines as limiting cases). Thus the locus in the w plane associated with the sliding short is a circle, as shown in Fig. 3, whose radius and center will be denoted by R and R_c, respectively, although these are, as yet, unknown. Together with w, however, they satisfy the equation

$$|w - R_c|^2 = R^2. \tag{12}$$

Next, w is eliminated between (12), (5), and (6). This is most conveniently done by noting that (12) is of the same form as (7). Thus the desired result may be obtained from (8) by substituting R_c and R for w_2 and $\zeta P_6/P_4$, respectively. This yields

$$A\left(\frac{P_3}{P_4}\right)^2 + 2B\left(\frac{P_3 P_5}{P_4^2}\right) + C\left(\frac{P_5}{P_4}\right)^2 + 2D\left(\frac{P_3}{P_4}\right) + 2E\left(\frac{P_5}{P_4}\right) + F = 0 \tag{13}$$

where

$$A=a \quad (14)$$
$$B=\zeta(c-a-b)/2 \quad (15)$$
$$C=\zeta^2 b \quad (16)$$
$$D=[R^2(b-a-c)+a(a-b-c)]/2 \quad (17)$$
$$E=\zeta[R^2(a-b-c)+b(b-a-c)]/2 \quad (18)$$
$$F=[R^4+R^2(c-a-b)+ab] \quad (19)$$

and where

$$a=|w_1-R_c|^2 \quad (20)$$
$$b=|R_c|^2 \quad (21)$$
$$c=|w_1|^2 \quad (22)$$
$$\zeta=\frac{1}{|K|^2}. \quad (23)$$

It can be shown that (13) represents an ellipse in the P_3/P_4, P_5/P_4 plane. Moreover, it is linear in $A \cdots F$. Substitution of the observed P_3/P_4, P_5/P_4, which correspond to the sliding-short positions, into (13) thus leads to a system of linear equations which may be solved for $A/F \cdots E/F$. Although a minimum of five positions is required, ordinarily a larger number is desirable, and a standard least squares solution is useful. The next task is to solve (14)$\cdots$(19) for a, b, c, ζ, and R^2.

Although (14)$\cdots$(19) represent a simultaneous set of six equations in a, b, c, ζ, and R^2, some of which are of the third degree, the remarkable result is that they can be solved in closed form! As an intermediate step, let

$$\alpha=(R^2+a)/\zeta \quad (24)$$
$$\beta=[(R^2-a)(R^2-b)+2R^2c]/\zeta \quad (25)$$
$$\gamma=R^2+b \quad (26)$$
$$\delta=(R^2-a)/\zeta \quad (27)$$
$$\epsilon=(R^2-b). \quad (28)$$

By substitution, it can be shown that

$$\frac{BD-AE}{AC-B^2}=\alpha \quad (29)$$
$$\frac{DE-BF}{AC-B^2}=\beta \quad (30)$$
$$\frac{BE-DC}{AD-B^2}=\gamma \quad (31)$$
$$\frac{AF-D^2}{AC-B^2}=\delta^2 \quad (32)$$
$$\frac{CF-E^2}{AC-B^2}=\epsilon^2. \quad (33)$$

Moreover, while the left-hand members of (29)–(33) are written as shown for convenience, inspection shows that they involve only the ratios $A/F \cdots E/F$ which, by hypothesis, have been determined from the sliding-short data as outlined above.

The solution to (24)–(28) is straightforward and may be written

$$a=|w_1-R_c|^2=\frac{(\alpha+\delta)(\gamma-\epsilon)}{2(\alpha-\delta)} \quad (34)$$
$$b=|R_c|^2=\frac{\gamma+\epsilon}{2} \quad (35)$$
$$c=|w_1|^2=\frac{\beta-\delta\epsilon}{\alpha-\delta} \quad (36)$$
$$\zeta=\frac{\gamma-\epsilon}{\alpha-\delta} \quad (37)$$
$$R^2=\frac{\gamma-\epsilon}{2}. \quad (38)$$

Although closed expressions for a, b, c, ζ, and R^2 have now been obtained, a problem remains. While α, β, and γ are uniquely determined by (29)–(31), only the squares of δ and ϵ may be obtained from (32) and (33). Their sign is still undetermined. Accordingly, depending upon how these signs are chosen, four sets of values for $a \cdots R^2$ are obtained, only one of which represents the desired solution. Recalling the definition of b and c from (21) and (22) and comparison with (27) and (28) indicates that the sign ambiguity centers around the questions as to whether the points w_1 and/or the origin are enclosed by the circle. In the example of Fig. 3, the origin lies inside while w_1 lies outside the circle. Apart from the sign ambiguity associated with δ and ϵ, the parameters R, R_c, ζ, and w may be obtained from (34)–(38) except for the sign of $\mathrm{Im}(R_c)$. It can be shown, however, that it is possible to assign the labels P_3, P_5, etc., to the detectors in such a way that a positive value is obtained. This will be assumed in what follows.

In addition to determining the proper choice of signs for δ and ϵ, the parameters $|M|^2$ and w_2 have yet to be determined. With regard to the multiple-root problem, the existence of the fourth detector (P_6) was ignored in (13) and the development that followed. The discussion thus pertains to a five- rather than six-port. Although the future for the five-port appears limited, it is worth noting that apart from the multiple-root problem, the foregoing, together with certain observations to follow, provide for its calibration. In the five-port mode, one ordinarily requires [1] that in Fig. 3 the design be such that the circle and the line between the origin 0 and w_1 do not intersect. This clearly places 0 and w_1 outside the circle and provides a basis for the choice of signs. One possible way of dealing with the ambiguity is thus to place sufficiently stringent specifications on the subcomponents from which the five- or six-port is constructed such that the desired result is assured.

Another straightforward experimental procedure for determining if 0 and/or w_1 is inside the circle is to determine whether, for some choice of passive termination at port 2, P_3 or P_5 can be made to vanish. Ordinarily, this is done by placing an attenuator of low residual loss ahead of the moving short. First, the short position is adjusted for a minimum value of P_3 (or P_5). Next, the attenuation is increased and the effect on P_3 (or P_5) noted. If 0 (or w_1) is inside the circle, there will be some choice of short and attenuator positions which will cause P_3 (or P_5) to go to 0. If this result is not obtained, the points lie outside the circle.

This approach, however, while of value in some circumstances, requires too much operator effort to be considered useful in a multiple-frequency environment. An alternative method will now be developed.

Thus far no use has been made of P_6 other than its role in obtaining (8). Unfortunately, the latter sheds little or no light on the immediate question. Alternatively, P_6 could be substituted for P_5 in (13) and the development which follows. Unfortunately, this yields two more sign ambiguities, namely, whether $|w_2|^2$ is greater or smaller than R^2, and whether arg w_2 is greater or smaller than arg R_c. These problems are avoided in the procedure to follow.

Let

$$w = u + jv \tag{39}$$

$$w_2 = u_2 + jv_2. \tag{40}$$

With these substitutions, (7) may be expanded to yield

$$s + 2uu_2 + 2vv_2 + \rho\frac{P_6}{P_4} = u^2 + v^2 \tag{41}$$

where

$$s = -\left(u_2^2 + v_2^2\right) \tag{42}$$

and

$$\rho = \frac{1}{|M|^2}. \tag{43}$$

If the relationship (42) among s, u_2, and v_2 is ignored for the moment, (41) may be regarded as a linear equation in s, u_2, v_2, and ρ which represent the parameters associated with P_6 and which are yet to be found. Given a set of values for P_6/P_4 and the corresponding values of u and v, it would be possible to form a system of linear equations from (41) which could be solved to yield s, u_2, v_2, and ρ. In order for this scheme to work, two conditions must be satisfied. First, one must be able to obtain the u and v corresponding to the observed P_6/P_4; second, the set of equations so formed must be linearly independent.

The u and v associated with the sliding-short positions may be found as follows. The procedure is illustrated in Fig. 3. Again, it will be assumed that the proper choices of sign have been made so that $|R_c|$, $|w_1|$, $|w_1 - R_c|$, R, and ζ are known. The argument of R_c will be denoted by ψ, and, as noted earlier, it will be assumed that $0<\psi<\pi$. Moreover, ψ may be found from $|R_c|$, $|w_1|$, and $|w_1 - R_c|$ by use of the law of cosines. In a similar way, u is given by

$$u = \frac{|w|^2 + |w_1|^2 - |w - w_1|^2}{2|w_1|} \tag{44}$$

and where, in accordance with the foregoing definitions,

$$|w|^2 = \frac{P_3}{P_4} \tag{45}$$

and

$$|w - w_1|^2 = \zeta\frac{P_5}{P_4}. \tag{46}$$

Apart from a sign ambiguity, v could now be found by use of the Pythagorean theorem. The following alternate approach avoids the sign problem. From Fig. 3, it is evident that

$$v = |w| \sin\theta \tag{47}$$

while, by use of a common trigonometric identity,

$$\sin\theta = \frac{\cos(\psi - \theta) - \cos\psi\cos\theta}{\sin\psi} \tag{48}$$

so that by further use of the law of cosines, (47) becomes

$$v = \frac{|w|^2 + |R_c|^2 - R^2 - 2uR_{cx}}{2R_{cy}} \tag{49}$$

where R_{cx} and R_{cy} are the real and imaginary parts of R_c.

By use of (44) and (49), the values of u and v for each of the sliding-short positions may be determined. These together with the observed P_6/P_4 values may be substituted in (41) to yield a linear system of equations in s, u, v, and ρ. Unfortunately, however, these equations are not linearly independent; in order to obtain an independent set, one must include one or more of the sliding-load observations. Here u is still given by (44), but (49) cannot be used for v since both R and R_c are unknown. Instead, v is obtained from

$$v = \sqrt{|w|^2 - u^2}. \tag{50}$$

This assumes, however, that the entire circle, or at least those portions thereof which correspond to the sliding-load data, lies on the same side of the real axis as R_c. Ordinarily, this assumption is well satisfied by practical six-port designs.

Although a minimum of three positions from the sliding-short data and one from the sliding-load data are required to form the set, it may be desirable to form more equations and to use a standard least squares solution. In any event, it is now possible to solve the system for s, u, v, and ρ.

In obtaining this result, however, it has been assumed that $|R_c|$, $|w_1 - R_c|$, R, and ζ are known, while in reality one has four sets of possible values for these parameters. The key point is now the following. If, and only if, the proper set of values has been chosen, the s, u, and v obtained above will satisfy (42).[4] The procedure is thus to obtain the s, u, and v corresponding to each of the four sets of values for $|R_c| \cdots \zeta$ and then test them against (42). In theory, it would only be necessary to continue this procedure until a set was found which satisfied (42); because of inevitable measurement errors, however, it is desirable to test all four sets of values and then retain the one which best satisfies (42). The five parameters that

[4] Although an analytical proof is lacking, this conclusion has been demonstrated repeatedly on a numerical basis.

describe the mapping from the P_3/P_4, P_5/P_4, P_6/P_4 space to the w plane have now been completely determined.

III. A Further Modification to the Procedure

As noted in earlier papers and implicit in the foregoing, the use of four detectors results in an over-determined system. The redundancy is exhibited in Fig. 3, for example, where w is determined to the extent of a choice between it and its conjugate by $|w|$ and $|w-w_1|$ (or P_3/P_4, P_5/P_4) alone. One of the more interesting challenges of the six-port technique is to exploit this redundancy feature or to make certain that optimal use has been made of the available information. The calibration procedure just described, while correct in principle, is subject to further improvement.

Perhaps the most obvious weakness of the foregoing technique is that three of the four detectors (P_3, P_4, and P_5) were initially singled out for preferred treatment in the five-port mode. While this permitted a determination of $|w_1|$, R_c, ζ, and R^2, the experimental error in obtaining these parameters is propagated to the subsequent determination of w_2 and ρ. As a rule, one would like a "symmetric" approach where all observations are given equal weight.

One possible way of restoring the symmetry would be to repeat the procedure for each of the three remaining possible combinations of the four detectors and then to average the results. Ordinarily, this does not appear desirable due to the computational time involved and because the solution may be ill-conditioned for certain detector combinations.

Alternatively, one may use the solution thus obtained as the starting point for an iterative solution to the set of cubic equations which, as discussed earlier, can be obtained from (8). While this effectively "erases" the preferential treatment given P_3, P_4, and P_5, it also discards the information that the sliding-short data lie in a circle in the w plane. Although the experimental evidence to date suggests that this modification does ordinarily yield an improvement in the calibration results, further study will be required to confirm this.

As noted earlier, an alternative procedure for obtaining an initial starting point for a solution based on (8) is merely to observe the collection P_3/P_4, P_5/P_4, P_6/P_4 for nine or more arbitrary terminations and then to solve a linear system of nine equations in nine unknowns. From this, an initial approximation to the paraboloid parameters may be obtained. Although this has the advantage of avoiding the multiple-root problems, it appears that in order to assure a well-conditioned solution to this linear system, it would be necessary to stipulate further that the "weakly reflecting" sliding terminations in fact have a substantial reflection (e.g., $|\Gamma|\sim 0.3-0.5$). The achievement of this on a broadband basis might prove difficult in practice.

This problem is avoided in the solution described above; moreover, it is applicable to both the five-port and six-port. In any event, however, both methods appear to be viable solutions to the problem. A more definitive statement of their relative merit must await further practical experience in their use.

IV. Determination of w

To complete the picture, it is necessary to obtain an explicit expression for $w=f(P_3,P_4,P_5,P_6)$. Returning to Figs. 2 and 3, if w_2 and $\sqrt{\rho P_6/P_4}$ are substituted for R_c and R, respectively, this problem is equivalent to finding the w associated with the sliding-short positions, which has already been solved. Making the appropriate substitutions in (44) and (49) one has

$$u=\frac{P_3/P_4-\zeta P_5/P_4+|w_1|^2}{2|w_1|} \tag{51}$$

$$v=\frac{P_3/P_4-\rho P_6/P_4+|w_2|^2-2uu_2}{2v_2}. \tag{52}$$

This assumes, however, that the three circles intersect in a point, but, because of measurement error, this will only be approximately true. The treatment of this problem and a more complete development of the theory contained herein will be found in another paper by the author [5].

V. Summary

This paper sketches the mathematical basis for and describes an experimental procedure which permits the six-port calibration problem to be reduced to that of a four-port. Following this, the same experimental results may be used to calibrate the reduced four-port [4]. A more complete treatment and theoretical development of this and related problems are given in a related paper by the author [5].

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